Snow Hydrology: The parameterization of subgrid processes within a physically based snow energy and mass balance model

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Project Summary

This is a summary of work proposed that will be conducted during the period 1998 to 2001. The objective of this research is to develop techniques for the representation and parameterization of subgrid and distributed snow processes within snowmelt models. Snowmelt is driven by energy exchanges at the snow surface that have variability down to scales of 1 to 10 m. When areas with large extent are modeled it is impractical to apply models distributed on a 1 to 10 m grid. Larger model elements, either grid squares, or topographically delineated need to be used. Therefore it is necessary to develop modeling approaches that can parameterize the subgrid variability within these elements. Due to nonlinearities and threshold processes involved in snowmelt the response over large model elements cannot be simply represented using 'effective' parameters. The goal here is to combine physically based modeling emphasizing physical understanding of the reasons for and processes involved in spatial variability of snow and snowmelt with analysis of extensive existing remotely sensed and field based data from two Colorado front range watersheds. We will use a fine scale distributed model to quantify and refine our understanding of the spatial snow accumulation and melt processes. This will then form the basis for parameterization of subgrid variability through the use of depletion curves and the derivation of these depletion curves from digital elevation data. The work proposed consists of:

(1) Calibration and validation of the distributed snowmelt model for the study watersheds so that it can serve as an encoding of our understanding of the spatially distributed snow processes for these watersheds.
(2) Reformulation of the point energy and mass balance snowmelt model to account for subgrid variability through the use of depletion curves. The depletion curve is used as a parameterization for the spatial distribution of snow within model elements. This will allow application of the model to larger elements.
(3) Use the fine scale distributed data and model of task 1 to develop depletion curves for areas typical of broad scale model elements, and attempt to establish relationships between depletion curves and digital elevation data, recognizing that topography is the primary physical determinant of spatial variability.
(4) Quantify the scale dependence of the depletion curves. Since the variability within model elements depends upon scale (size) so should the depletion curves. We will attempt to develop general scaling rules for working with depletion curves in the context of the physically based model that is being used.
(5) Implementation of findings and results into appropriate National Operational Hydrologic Remote Sensing Center (NOHRSC) operational systems.

Statement of the problem

Snowmelt is a significant surface water input of importance to many aspects of hydrology and water resources management. Snowmelt is primarily driven by energy exchanges at the snow surface that have variability down to scales of 1 to 10 m. The physical processes responsible for snowmelt at these point scales are relatively well understood and modeled by a variety of 'point' models (Anderson, 1976; Morris, 1986; Morris, 1990; Jordan, 1991; Tarboton et al., 1995; Tarboton and Luce, 1996). However when larger scales are considered it is frequently computationally prohibitive or there is insufficient data to apply such models in a distributed fashion at each grid point with 1 to 10 m spacing as would be required for true physical representation of the process. Therefore it is necessary to develop modeling approaches that can parameterize the subgrid variability. This work addresses the need for a physically based understanding of subgrid variability of snow processes so that an appropriate physically based parameterization for use with physically based snowmelt models can be developed.

Data

Our approach involves the combination of modeling with analysis of remotely sensed and field data. The region selected is in the Colorado Front Range just west of Boulder, Colorado. Two basins within this region will be used for intensive analysis and model development. Green Lakes Valley (GLV) is an 8-km^2 alpine basin where intensive snow surveys were carried out on two dates near maximum snow accumulation in 1996. More than 550 depth measurements and 17 snow pits were excavated and used to grid snow depth and density at 50, 100 and 200 m spacing. Loch Vale Watershed (LVWS) was also sampled in 1996 more for test and validation purposes with reduced effort. Snow depth and water equivalence was sampled at 13 carefully located 100 x 100 m ‘pixels’ using a total of 109 depth measurements and 39 density measurements.

Micrometeorological data from four stations in GLV and three stations in LVWS ranging from valley bottom to ridge top locations has been collected. This comprises air temperature, relative humidity, wind speed and direction and incident solar radiation. Two sites (one in each watershed) also collect additional wind profile data and radiation data including upward and downward longwave radiation. Precipitation is measured at two sites in GLV and all three sites in LVWS. Snow depth is measured acoustically at one site in each watershed.

Multi-platform, multi-resolution, multi-temporal remote sensing data has been collected for the 1996 snow season. These data comprise:

- Metric 1:24,000 aerial photography on seven dates. These have been scanned and high resolution (~1 m) orthophotos produced.
- Airborne Visible and Infrared Imaging Spectrometer (AVIRIS) and coincident high altitude color-IR aerial photographs and hyperspectral data on three dates.
- NOAA Advanced Very High resolution Radiometer (AVHRR) and Geostationary Operational Environment (GOES) satellite images classified into areal extent of snow cover maps on 40 dates.
- Digital elevation data at 10 m resolution produced photogrametrically.

In addition to this we plan to acquire Landsat Thematic Mapper (TM) data for four dates to obtain snow cover fraction maps using the Rosenthal and Dozier (1996) algorithm.

Tasks

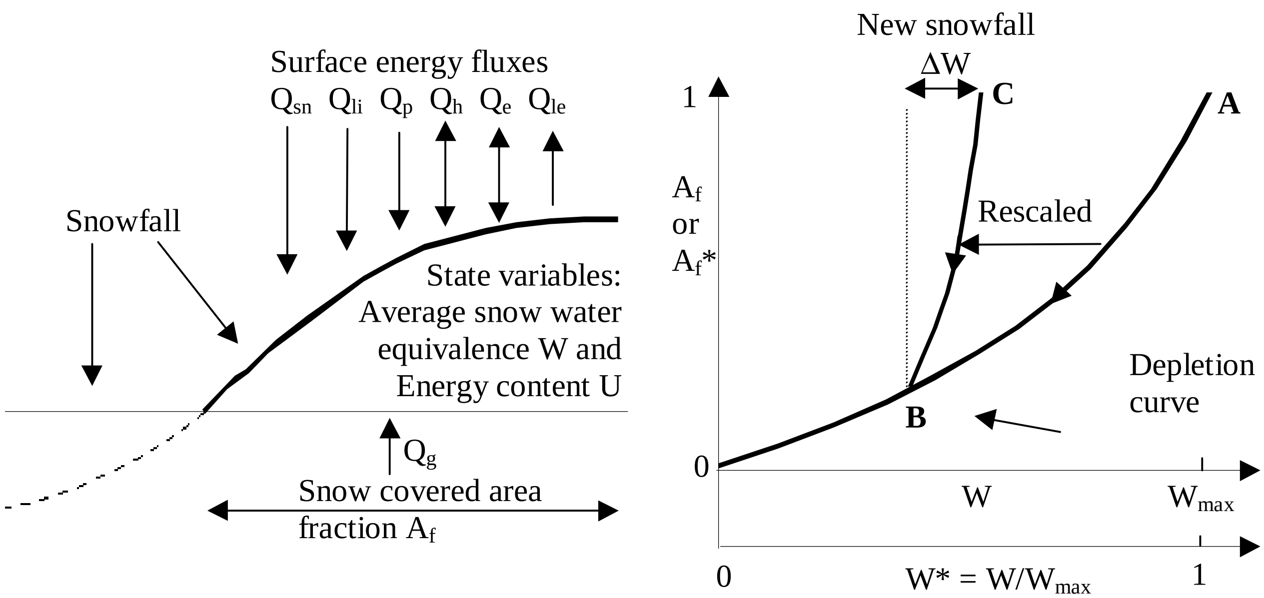
Tasks 1 and 2 involve the subgrid parameterization of the point scale snowmelt model. In task 1, the fine scale model, model elements are essentially points on a fine grid (~ 10m) over the watershed. However in task 2 we are developing a subgrid parameterization to be used with much larger model elements, say 0.2 km^2 and upwards, even potentially to land-atmosphere model scale. The model elements may be irregularly shaped if for example they are subwatersheds or hydrologic response units (e.g. Leavesley et al., 1983) demarcated based upon surface topography, or they may be square or rectangular if a broad (coarse) scale grid is being used. Task 3 involves new theory for the development of depletion curves. Task 4 addresses scale issues and task 5 is operational implementation.

Task 1. Validation of fine scale distributed snowmelt model against GLV and LVWS data.
We will apply the Utah Energy Balance (UEB) snowmelt model (Tarboton et al., 1995; Tarboton and Luce, 1996) (see http://www.engineering.usu.edu/dtarb) model at fine scale (~10 m grid) over the GLV and LVWS watersheds for the time periods that data is available. Model changes will include:

- Calibration of drift factors appropriate for this region.
- Parameterization of vegetation density and roughness parameters based on remotely sensed data related to vegetation (AVIRIS data).

The model will be driven by the micrometeorological. The model will be tested against snow depth and water equivalence measurements. The model will also be tested against spatially distributed patterns from remote sensing of snow cover and snow cover fractions after aggregating up to comparable scales (30 m for Landsat TM and 1 km for AVHRR snow cover). The calibrated distributed model will serve as an encoding of our understanding of the spatially distributed snow process.

Task 2. Generalization of snow model to use depletion curves and include snow covered area as a state variable.
Figure 1 depicts schematically the modifications to the UEB model that will be implemented to parameterize subgrid variability using depletion curves.



<u>Figure 1.</u> Schematic of revised model.

The snow covered area fraction, A_f, is introduced as a new state variable. The point snowmelt model is then used to calculate fluxes to and from this fractional area, which is adjusted after each time step, based on changes in W, which here denotes the total or model element average snow water equivalence. During melting as W decreases, A_f is decreased following along the depletion curve. When there is new snowfall, W is incremented by the new snowfall water equivalence ΔW (taken over the whole area) and A_f goes to one. The new snowfall (covering the whole element) will be subjected to the same processes that lead to spatial variability as the old snow. Also, it will melt first. Therefore we assume the system returns along a rescaled depletion curve to the point of original departure.

Now the rate of progress along a depletion curve, which is controlled by its steepness, depends upon the amount of snow present. Therefore we propose a single dimensionless depletion curve, scaled by the maximum snow water equivalence (so far) for the season (or since W was last 0). This will provide automatic scaling of the depletion curve letting the onset of melt be determined naturally from the modeling of physical processes, rather than parameters determining the ‘beginning’ of the melt season as is the case for some index models. The following equation gives a particular depletion curve, $A_f(W)$, in terms of the dimensionless depletion curve.

$$A_f(W) = A_f^*(W / W_{max}) \qquad (1)$$

In a preliminary study (Luce et al., 1997) observations and modeling were used to empirically describe the depletion curve and model basinwide integrated snowmelt following this approach with encouraging results (Figure 2).

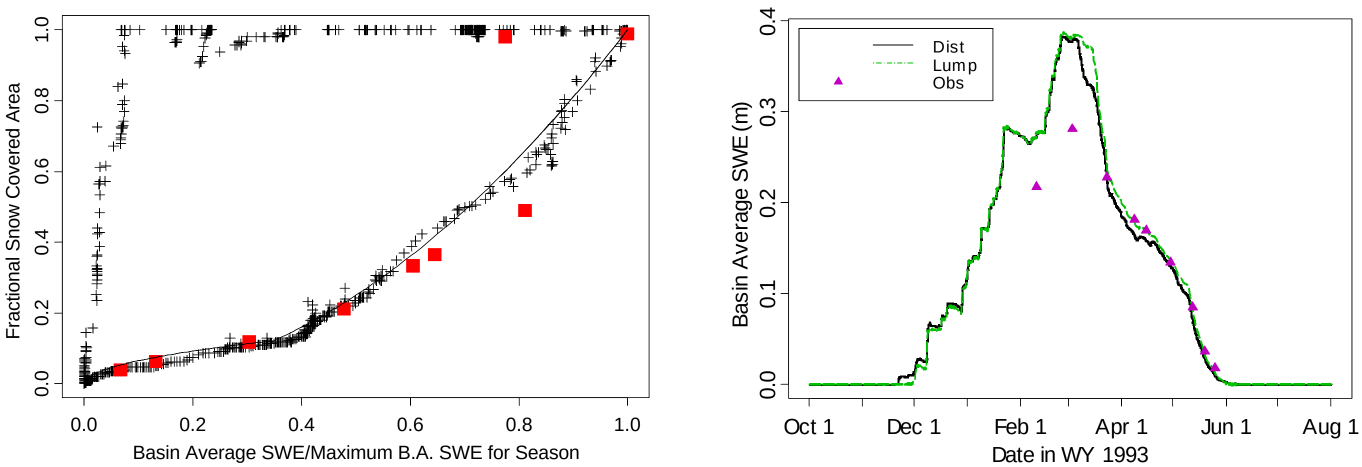


Figure 2. a) Depletion curve developed from fine scale (30 m grid) UEB model application at Upper Sheep Creek. Crosses are derived from the fine scale model and squares are observed data. b) Comparison of snow water equivalence from fine scale and lumped (treating 0.26 km^2 watershed as single model element) model runs. Preliminary results presented by Luce et al. (1997).

Task 3. Development of depletion curves.

In figure 2 we gave a preliminary example of a depletion curve developed from a distributed model application at Upper Sheep Creek, within the Reynolds Creek Experimental watershed. This enabled the 0.26 km^2 watershed that was modeled using 255 distributed ‘point’ model elements to be modeled as a single model element, with minimal loss of accuracy in terms of overall snow water equivalence. In this work we intend to do the same for the GLV and LVWS study areas. Since GLV and LVWS are larger (8 and 6.6 km^2 versus 0.26 km^2) separate depletion curves will be developed for several sub areas within these watersheds, using the validated distributed model as an interpolator to provide the means to estimate snow water equivalence at any point or over any area or subwatershed at any time. Results from the distributed model will be used to parameterize areal depletion curves and relate these to quantifiable DEM based variables, such as

- Elevation distribution
- Distribution of exposures to shortwave and longwave radiation (for example using TOPORAD Dozier, 1979). This will capture slope/aspect/shadowing effects.

Since one way or the other (through exposure to radiation, or elevation and temperature) topography is a primary physical determinant of much spatial variability in snow accumulation and melt, we will explore the possibility of generating depletion curves from topography and topographically derived variables (such as integrals of radiation exposure over time) alone.

The theoretical basis for this idea is in the link between the areal depletion curve and the snow water equivalence distribution function. Since the areal depletion curve represents the functional decrease of snow covered area fraction, A_f, with decreasing model element average W, it can be viewed as a parameterization of the distribution of snow over the model element. An incremental reduction in W results in an incremental reduction in snow covered area leading to the interpretation that the snow water equivalence on that fraction of the area was less than or equal to the incremental reduction (melt) amount. To formally develop this relationship assume a generic probability distribution (pdf) for snow water equivalence, $f_g(w)$, that retains a consistent shape. This pdf gives the probability for point snow water equivalence areally sampled, offset by an additive constant. This is shown schematically in figure 3.

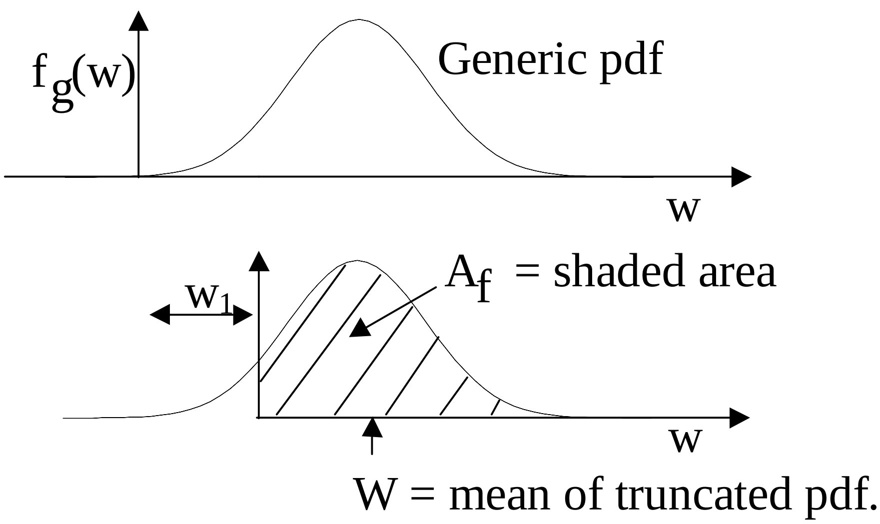


Figure 3. Schematic of generic snow water equivalence probability distribution.

As the snow accumulates and ablates this function shifts to the left or right. The positioning of the generic pdf is controlled by the parameter w_1. The tail to the left of the y axis represents snow free area, for any particular w_1. The snow covered area fraction is therefore, in terms of this pdf, defined as:

$$A_f(w_1) = \int_0^\infty f_g(w + w_1)dw = \int_{w_1}^\infty f_g(w)dw \qquad (2)$$

The probability distribution of snow water equivalence, for any particular w_1 has a nugget at zero, i.e. the finite probability of the areally sampled snow water equivalence being zero is $1\text{-}A_f(w_1)$. The part of the pdf to the right of the axis represents the snow water equivalence pdf for non zero snow water equivalences, i.e. snow covered points in the areal sampling. Consequently the element average snow water equivalence is defined (from the usual definition of a mean) as:

$$W(w_1) = \int_0^\infty wf_g(w + w_1)dw = \int_{w_1}^\infty (w - w_1)f_g(w)dw = \int_{w_1}^\infty wf_g(w)dw - w_1A_f = \int_{w_1}^\infty A_f(w)dw \quad (3)$$

In the above both A_f and W are functions of the positioning parameter w_1. The last step in (3) is obtained through integration by parts. Equation (2) establishes $A_f(w)$ as a cumulative exceedence function of the generic pdf, $f_g(w)$. Note that the functional argument 'w' is a point snow water equivalence, different from the model element average snow water equivalence, denoted using capital ‘W’. Furthermore it is also possible, given the depletion curve, $A_f(W)$, and assuming the existence of a consistent generic pdf, $f_g(w)$, to compute $f_g(w)$, as follows. Differentiating equations (2) and (3) gives

$$\frac{dA_f}{dw_1} = -f_g(w_1) \tag{4}$$

$$\frac{dW}{dw_1} = -A_f(w_1) \tag{5}$$

Combining (dividing) these yields

$$\frac{dA_f}{dW} = \frac{f_g}{A_f} \quad \text{or} \quad f_g = A_f \frac{dA_f}{dW} \tag{6}$$

This establishes values for the generic pdf as a function of A_f or W, but not yet the independent variable w_1 it is evaluated at. w_1 is obtained by integrating (4), starting from the boundary condition that when w_1=0, A_f=1 (see figure 3).

$$w_1 - 0 = \int_{A_f=1}^{A_f} -\frac{1}{f_g} dA_f \quad \text{or} \quad w_1 = \int_{A_f}^{1} \frac{1}{f_g} dA_f \tag{7}$$

Equations (6) and (7) establish the mathematical inverse of (2) and (3), linking the snow water equivalence pdf, $f_g(w)$, and depletion curve $A_f(W)$. In practice the depletion curve is most likely to be expressed numerically as a series or table of data values, so these derivatives and integrals will need to be evaluated numerically. These equations establish a procedure that can (at least numerically) from a generic $f_g(w)$ determine the depletion curve $A_f(W)$ and vise versa.

This theory offers the potential for predicting the depletion curve for an ‘unsampled’ basin from topography and vegetation. In this task we will see if the generic distribution function, $f_g(w)$, can be constructed from combinations of topography and vegetation derived parameters that would be available in a geographic information system (GIS). Our approach will be to use the distributed data and fine scale model of Task 1 to provide us with a fine scale estimate of the actual snow distribution function. This will be related to the digital elevation model and other GIS data through the distribution of melt energy inputs and driving processes as modeled using the fine scale model.

Task 4. Exploration of scale issues.

The depletion curve concept is time and space scale dependent. We will try to quantify and better understand these scale dependencies through the development of rules or parameterizations for scaling depletion curves.

Task 5. Operational implementation.

Part of the mission of the National Operational Hydrologic Remote Sensing Center is the development and implementation of research and technology related to the use of remote sensing and modeling to provide better hydrologic forecasts. The generalized model to be developed fits into this realm and the results and findings will be evaluated, and where appropriate, transferred and implemented within their operational systems.

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